CHE374 – Economic Analysis & Decision Making (Homework Problem Set 1)

Question 1) (a)

Reactor	Tank Cost	Installation	Total
New Reactor	$8,000	$1,200	$9200
Old Reactor	$6,800	$2,500	$9300

Selling Old and Buying a New reactor is more profitable as it's a $100 cheaper.

Sell Old, Buy New = $9200 – $6,800 = $2,400
Use Old = 2,500 (installation) = $2,500

Question 1) (b) In order to be indifferent to the decision the Cost (Sell Old & Buy New) = Cost (install old)

$9200 – $(Old reactor Market value) = $2500

$6,700 = $9200 – $2500 = Old reactor market value.

Question 2) (a)

Company	Upfront Costs	Profit (Annually)
X	$ 7,500	$ 1700
Y	$ 9,000	$ 2,200

Let t = recovery time (of upfront costs)

$$t_x = \frac{\$7{,}500}{\$1{,}700} = 4.412 \text{ years}$$ (roughly 4.5 years to recover upfront costs) for company X

$$t_y = \frac{\$9000}{\$2{,}200} = 4.09 \text{ years}$$ (roughly 4 years to recover upfront costs) for company Y.

Question 2 (b) The two main issues with this type of comparisons is the lack of acknowledgement of the time horizon and time value of money.

In other words, we cannot directly compare the recovery costs, because it depends on the duration of the project (time horizon). Say for example the project runs for less then four years, then company Y provides a better offer and for greater then 4 years but less than 4.5 years company X provides a better offer.

Question 3) Definition of euler's number $e = \lim_{x \to 0} (1+x)^{1/x}$

Then $\lim_{m \to \infty} \left(1+\frac{r}{m}\right)^m$, As $m \to \infty$, $\frac{r}{m} \to 0$

$$\lim_{m \to \infty} \left[\left(1+\frac{r}{m}\right)^{\frac{m}{r}}\right]^r \to \left[\lim_{m \to \infty} \left(1+\frac{r}{m}\right)^{\frac{m}{r}}\right]^r = e^r$$

Question 4) $r_{eff} = 3.4\%$ per year. Find nominal annual interest

(a) Semi-annual compounding ($n_y = 2$, $n_m = 2$)

$$r_{eff} = 0.034 = \left(1 + \frac{r_{s/y}}{2}\right)^2 - 1$$

$$r_{s/y} = \left(\sqrt{0.034 + 1} - 1\right) \times 2 = 0.033716$$

$$\boxed{r_{s/y} = 3.372\%}$$

(b) Monthly Compounding ($n_y = 12$, $n_m = 12$)

$$r_{eff} = 0.034 = \left(1 + \frac{r_{m/y}}{12}\right)^{12}$$

$$r_{m/y} = \left(\sqrt[12]{1 + 0.034} - 1\right) \times 12 = 0.0334814$$

$$\boxed{r_{m/y} = 3.348\%}$$

(c) Daily Compounding ($n_y = 365$, $n_m = 365$)

$$r_{eff} = 0.034 = \left(1 + \frac{r_{d/y}}{365}\right)^{365}$$

$$r_{d/y} = \left(\sqrt[365]{1 + 0.034} - 1\right) \times 365 = 0.03343630$$

$$\boxed{r_{d/y} = 3.344\%}$$

(d) Continuous Compounding

$$r_{eff} = e^r - 1 \Rightarrow e^r = 1 + r_{eff} = 1.034$$

$$r_{cc} = \ln(1.034) = 0.03343477 \quad \boxed{r_{cc} = 3.343\%}$$

(e) How much money would be owed by the end of 4th year in each case?

$$\text{Amount owed} = 100{,}000 * (1 + r_{eff})^4$$
$$= 100{,}000 * (1 + 0.034)^4$$
$$\boxed{= \$114309.46}$$

Question 5) \$100 T-Bill, Annual Continuous Compounding rate of 2.3%

(a) Find effective annual rate assuming yearly compounding?

$$r_{eff,\,annual} = e^r - 1 = e^{0.023} - 1 = 0.0232665 \approx 2.327\%$$

(b) What is the price of the T-bill if it matures in 6 months?

$$P_0 = 100\, e^{-0.023(0.5)} = 98.8565 \approx \$98.86$$

(c) What is the price of the T-bill if it matures in 9 months?

$$P_0 = 100e^{-0.023(0.75)} = 98.2897 \approx \$98.29$$

(d) Continuous compounding rate drops to 2%.

$$P_0(6 \text{ months}) = 100\left(e^{-0.02\left(\frac{6}{12}\right)}\right) = 99.0049 \approx \$99.00$$

$$P_0(9 \text{ months}) = 100\left(e^{-0.02\left(\frac{9}{12}\right)}\right) = 98.51119 \approx \$98.51$$

Question 6) Assume 252 trading days per year.

(a) $r_{eff} = 8\%$ Find monthly interest based on monthly compounding

$n_y = 12$

$n_m = 1$

$$r_{eff} = \left(1+\frac{r_{m/m}}{1}\right)^{12} - 1 \rightarrow r_{m/m} = \sqrt[12]{1+0.08} - 1 = 0.006434$$
$$= 0.6434\%$$

(b) $r_{d/y} = 3.5\%$ Find the interest per year effective

daily compounding $n_y = 252$

$n_m = 252$

$$r_{eff} = \left(1+\frac{0.035}{252}\right)^{252} - 1 = 0.035617$$
$$\approx 3.562\%$$

(c) 4% interest per year with quaterly compounding. Find r_{cc} per year.

$$r_{eff} = e^{r} - 1 = \left(1+\frac{r_{q/y}}{4}\right)^4 - 1 \rightarrow r = \ln\left(1+\frac{0.04}{4}\right)^4 = 3.980\%$$

(d) 1.5% interest per month compounded monthly. Find r_{cc} per quarter.

$$r_{eff} = e^{4*r} - 1 = (1+0.015)^{12} - 1 \rightarrow r = \frac{1}{4}\ln(1+0.015)^{12} = 3\ln(1.015) = 4.467\%$$

(e) 1.2% interest per quarter with monthly compounding. Find the interest per three years based on semi-annual compounding.

$n_y = 12 \cdot 3$ (# of months in 3-years).

$n_m = 3$ (# months in a quarter)

$$r_{eff}(3 \text{ years}) = \left(1+\frac{0.012}{3}\right)^{36} - 1$$

$$r_{eff}(3 \text{ years}) = 0.1545522 4338$$

$$\underset{(3\text{ years})}{r_{eff}} = \left(1+\frac{r_{s/3y}}{6}\right)^6 - 1 = \left(\sqrt[6]{1.15455224338} - 1\right) \times 6$$

$$r_{s/3y} = 14.544\%$$